Chapter 9

Inferences Based on Two Samples

Confidence Intervals and Tests of Hypotheses

9.2 Assumptions about the two populations:

1. Both sampled populations have relative frequency distributions that are approximately normal.
2. The population variances are equal.

Assumptions about the two samples:

The samples are randomly and independently selected from the population.

9.4 The confidence interval for $(\mu_1 - \mu_2)$ is (–10, 4). The correct inference is **d** –no significant difference between means. Since 0 is contained in the interval, it is a likely value for $(\mu_1 - \mu_2)$. Thus, we cannot say that the 2 means are different.

9.6 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \Rightarrow (5,275 - 5,240) \pm 1.96\sqrt{\frac{150^2}{400} + \frac{200^2}{400}}$$
$$\Rightarrow 35 \pm 24.5 \Rightarrow (10.5, 59.5)$$

We are 95% confident that the difference between the population means is between 10.5 and 59.5.

b. The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(5275 - 5240) - 0}{\sqrt{\dfrac{150^2}{400} + \dfrac{200^2}{400}}} = 2.8$

The p-value of the test is $p = P(z \le -2.8) + P(z \ge 2.8) = 2P(z \ge 2.8) = 2(.5 - .4974)$
$= 2(.0026) = .0052$

Since the p-value is so small, there is evidence to reject H_0. There is evidence to indicate the two population means are different for $\alpha > .0052$.

c. The p-value would be half of the p-value in part **b**. The p-value $= P(z \ge 2.8) = .5 - .4974 = .0026$. Since the p-value is so small, there is evidence to reject H_0. There is evidence to indicate the mean for population 1 is larger than the mean for population 2 for $\alpha > .0026$.

d. The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(5275 - 5240) - 25}{\sqrt{\dfrac{150^2}{400} + \dfrac{200^2}{400}}} = .8$

The p-value of the test is $p = P(z \le -.8) + P(z \ge .8) = 2P(z \ge .8) = 2(.5 - .2881)$
$= 2(.2119) = .4238$

Since the p-value is so large, there is no evidence to reject H_0. There is no evidence to indicate that the difference in the 2 population means is different from 25 for $\alpha \le .10$.

e. We must assume that we have two independent random samples.

9.8 a. $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \dfrac{(25-1)200+(25-1)180}{25+25-2} = \dfrac{9120}{48} = 190$

b. $s_p^2 = \dfrac{(20-1)25+(10-1)40}{20+10-2} = \dfrac{835}{28} = 29.8214$

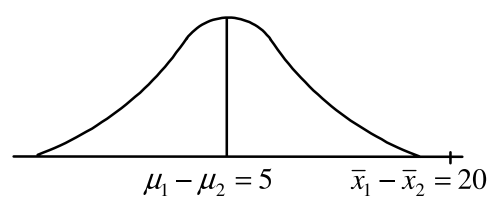
c. $s_p^2 = \dfrac{(8-1).20+(12-1).30}{8+12-2} = \dfrac{4.7}{18} = .2611$

d. $s_p^2 = \dfrac{(16-1)2500+(17-1)1800}{16+17-2} = \dfrac{66,300}{31} = 2138.7097$

e. s_p^2 falls nearer the variance with the larger sample size.

9.10 a. $\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}} = \sqrt{\dfrac{100}{100} + \dfrac{64}{100}} = \sqrt{1.64} = 1.2806$

b. The sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal by the Central Limit Theorem since $n_1 \ge 30$ and $n_2 \ge 30$.



c. $\bar{x}_1 - \bar{x}_2 = 70 - 50 = 20$

Yes, it appears that $\bar{x}_1 - \bar{x}_2 = 20$ contradicts the null hypothesis H_0: $\mu_1 - \mu_2 = 5$.

d. The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

e. H_0: $\mu_1 - \mu_2 = 5$
H_a: $\mu_1 - \mu_2 \ne 5$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 5}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(70-50)-5}{1.2806} = 11.71$

The rejection region is $z < -1.96$ or $z > 1.96$. (Refer to part **d**.)

Since the observed value of the test statistic falls in the rejection region (z = 11.71 > 1.96), H_0 is rejected. There is sufficient evidence to indicate the difference in the population means is not equal to 5 at $\alpha = .05$.

f. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table IV, Appendix A, $z_{.025}$ = 1.96. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \Rightarrow (70 - 50) \pm 1.96\sqrt{\frac{100}{100} + \frac{64}{100}} \Rightarrow 20 \pm 2.51 \Rightarrow (17.49,\ 22.51)$$

We are 95% confident the difference in the two population means ($\mu_1 - \mu_2$) is between 17.49 and 22.51.

g. The confidence interval for $\mu_1 - \mu_2$ gives more information than the test of hypothesis. In the test, all we know is that $\mu_1 - \mu_2 \neq 5$, but not what $\mu_1 - \mu_2$ might be. With the confidence interval, we do have a range of values that we believe will contain $\mu_1 - \mu_2$.

9.12 a. Some preliminary calculations are:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(17 - 1)3.4^2 + (12 - 1)4.8^2}{17 + 12 - 2} = 16.237$$

To determine if there is a difference in the means of the 2 groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{(5.4 - 7.9) - 0}{\sqrt{16.237\left(\dfrac{1}{17} + \dfrac{1}{12}\right)}} = -1.646$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t distribution with df = $n_1 - n_2 - 2$ = 17 + 12 – 2 = 27. From Table VI, Appendix A, $t_{.025}$ = 2.052. The rejection region is $t < -2.052$ or $t > 2.052$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.646 \not< -2.052$), H_0 is not rejected. There is insufficient evidence to indicate the 2 means are different at $\alpha = .05$.

b. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table VI, Appendix A, with df = $n_1 + n_2 - 2$ = 17 + 12 – 2 = 27, $t_{.025}$ = 2.052. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.025}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} \Rightarrow (5.4-7.9) \pm 2.052\sqrt{16.237\left(\frac{1}{17}+\frac{1}{12}\right)}$$
$$\Rightarrow -2.50 \pm 3.12 \Rightarrow (-5.62,\ 0.62)$$

We are 95% confident that the true difference in means is between –5.62 and 0.62.

9.14 a. Let μ_1 = mean time on the Trail Making Test for schizophrenics and μ_2 = mean time on the Trail Making Test for normal subjects. The parameter of interest is $\mu_1 - \mu_2$.

b. To determine if the mean time on the Trail Making Test for schizophrenics is larger than that for normal subjects, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 > 0$

c. Since the p-value is less than α ($p = .001 < .01$), H_0 is rejected. There is sufficient evidence to indicate the mean time on the Trail Making Test for schizophrenics is larger than that for normal subjects at $\alpha = .01$.

d. For confidence coefficient .99, $\alpha = .01$ and $\alpha/2 = .005$. From Table IV, Appendix A, $z_{.005} = 2.58$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.005}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \Rightarrow (104.23-62.24) \pm 2.58\sqrt{\frac{45.45^2}{41}+\frac{16.34^2}{49}}$$
$$\Rightarrow 41.99 \pm 19.28 \Rightarrow (22.71,\ 61.27)$$

We are 99% confident that the true difference in mean time on the Trail Making Test between schizophrenics and normal subjects is between 22.71 and 61.27.

9.16. a. Let μ_1 = mean IBI value for the Muskingum River Basin and μ_2 = mean IBI value for the Hocking River Basin. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The 90% confidence interval is:

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{.05}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \Rightarrow \left(.035-.340\right) \pm 1.645\sqrt{\frac{1.046^2}{53}+\frac{.960^2}{51}}$$
$$\Rightarrow -.305 \pm .324 \Rightarrow (-.629,\ .019)$$

We are 90% confident that the difference in mean IBI values between the two river basins is between −.629 and 0.019.

b. To determine if the mean IBI values differ for the two river basins, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(.035 - .340) - 0}{\sqrt{\dfrac{1.046^2}{53} + \dfrac{.96^2}{51}}} = -1.55$

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.55 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean IBI values differ for the two river basins at $\alpha = .10$.

Both the confidence interval and the test of hypothesis used the same level of confidence – 90%. Both used the same values for the sample means and the sample variances. Thus, the results must agree. If the hypothesized value of the difference in the means falls in the 90% confidence interval, it is a likely value for the difference and we would not reject H_0. On the other hand, if the hypothesized value of the difference in the means does not fall in the 90% confidence interval, it is an unusual value of the difference and we would reject it.

9.18 a. Let μ_1 = mean trap spacing for the BT fishing cooperative and μ_2 = mean trap spacing for the PA fishing cooperative. The parameter of interest is $\mu_1 - \mu_2$.

b. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: BT, PA

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
BT	7	89.86	11.63	70.00	82.00	93.00	99.00	105.00
PA	8	99.63	27.38	66.00	76.50	96.00	115.00	153.00

The point estimate of $\mu_1 - \mu_2$ is $\bar{x}_1 - \bar{x}_2 = 89.86 - 99.63 = -9.77$.

c. Since the sample sizes are so small ($n_1 = 7$ and $n_2 = 8$), the Central Limit Theorem does not apply. In order to use the z statistic, both σ_1^2 and σ_2^2 must be known.

d. $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \dfrac{(7-1)11.63^2 + (8-1)27.38^2}{7 + 8 - 2} = 466.0917$

For confidence coefficient .90, $\alpha = .10$ and $\alpha / 2 = .05$. From Table VI, Appendix A, with $df = n_1 + n_2 - 2 = 7 + 8 - 2 = 13$, $t_{.05} = 1.771$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.05}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \Rightarrow (89.86 - 99.63) \pm 1.771\sqrt{466.0917\left(\frac{1}{7} + \frac{1}{8}\right)}$$

$$\Rightarrow -9.77 \pm 19.79 \Rightarrow (-29.56, 10.02)$$

We are 90% confident that the true difference in mean trap space between the BT fishing cooperative and the PA fishing cooperative is between -29.56 and 10.02.

e. Since 0 is contained in the confidence interval, there is no evidence to indicate a difference in mean trap spacing between the 2 cooperatives.

f. We must assume that the distributions of trap spacings for the two cooperatives are normal, the variances of the two populations are equal, and that the samples are random and independent.

9.20 $$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = \frac{(28-1)8.43^2+(32-1)9.56^2}{28+32-2} = 81.9302$$

The test statistic is $$t = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{82.64-84.81}{\sqrt{81.9302\left(\frac{1}{28}+\frac{1}{32}\right)}} = -.93$$

This agrees with the researcher's test statistic.

Using MINITAB with df = $n_1 + n_2 - 2 = 28 + 32 - 2 = 58$, the p-value is $p = P(t \le -.93) + P(t \ge .93) = 2(.1781) = .3562$. This is close to the p-value of .358. This agrees with the researcher's conclusion.

9.22 a. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Text-line, Witness-line, Intersection

Variable	N	Mean	Median	StDev	Minimum	Maximum	Q1	Q3
Text-lin	3	0.3830	0.3740	0.0531	0.3350	0.4400	0.3350	0.4400
Witness-	6	0.3042	0.2955	0.1015	0.1880	0.4390	0.2045	0.4075
Intersec	5	0.3290	0.3190	0.0443	0.2850	0.3930	0.2900	0.3730

Let μ_1 = mean zinc measurement for the text-line, μ_2 = mean zinc measurement for the witness-line, and μ_3 = mean zinc measurement for the intersection.

$$s_p^2 = \frac{(n_1-1)s_1^2+(n_3-1)s_3^2}{n_1+n_3-2} = \frac{(3-1).0531^2+(5-1).0443^2}{3+5-2} = .00225$$

For $\alpha = .05$, $\alpha/2 = .05/2 = .025$. Using Table VI, Appendix A, with df = $n_1 + n_3 - 2 = 3 + 5 - 2 = 6$, $t_{.025} = 2.447$. The 95% confidence interval is:

$$(\bar{x}_1-\bar{x}_3) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_3}\right)} \Rightarrow (.3830-.3290) \pm 2.447\sqrt{.00225\left(\frac{1}{3}+\frac{1}{5}\right)}$$
$$\Rightarrow 0.0540 \pm .0848 \Rightarrow (-0.0308,\ 0.1388)$$

We are 95% confident that the difference in mean zinc level between text-line and intersection is between -0.0308 and 0.1388.

To determine if there is a difference in the mean zinc measurement between text-line and intersection, we test:

H_0: $\mu_1 - \mu_3 = 0$
H_a: $\mu_1 - \mu_3 \neq 0$

The test statistic is $t = \dfrac{(\bar{x}_1 - \bar{x}_3) - D_o}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_3}\right)}} = \dfrac{(.3830 - .3290) - 0}{\sqrt{.00225\left(\dfrac{1}{3} + \dfrac{1}{5}\right)}} = 1.56$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t-distribution with df = $n_1 + n_3 - 2 = 3 + 5 - 2 = 6$. From Table VI, Appendix B, $t_{.025} = 2.447$. The rejection region is $t < -2.447$ or $t > 2.447$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.56 \not> 2.447$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean zinc measurement between text-line and intersection at $\alpha = .05$.

b. $s_p^2 = \dfrac{(n_2 - 1)s_2^2 + (n_3 - 1)s_3^2}{n_2 + n_3 - 2} = \dfrac{(6-1).1015^2 + (5-1).0443^2}{6 + 5 - 2} = .00660$

For $\alpha = .05$, $\alpha / 2 = .05 / 2 = .025$. Using Table VI, Appendix A, with df = $n_2 + n_3 - 2 = 6 + 5 - 2 = 9$, $t_{.025} = 2.262$. The 95% confidence interval is:

$$(\bar{x}_2 - \bar{x}_3) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_2} + \frac{1}{n_3}\right)} \Rightarrow (.3042 - .3290) \pm 2.262\sqrt{.00660\left(\frac{1}{6} + \frac{1}{5}\right)}$$
$$\Rightarrow -0.0248 \pm .1113 \Rightarrow (-0.1361,\ 0.0865)$$

We are 95% confident that the difference in mean zinc level between witness-line and intersection is between -0.1361 and 0.0865.

To determine if there is a difference in the mean zinc measurement between witness - line and intersection, we test:

H_0: $\mu_2 - \mu_3 = 0$
H_a: $\mu_2 - \mu_3 \neq 0$

The test statistic is $t = \dfrac{(\bar{x}_2 - \bar{x}_3) - D_0}{\sqrt{s_p^2\left(\dfrac{1}{n_2} + \dfrac{1}{n_3}\right)}} = \dfrac{(.3042 - .3290) - 0}{\sqrt{.00660\left(\dfrac{1}{6} + \dfrac{1}{5}\right)}} = -0.50$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the *t*-distribution with df $= n_2 + n_3 - 2 = 6 + 5 - 2 = 9$. From Table VI, Appendix B, $t_{.025} = 2.262$. The rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -0.50 \nless -2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean zinc measurement between witness-line and intersection at $\alpha = .05$.

c. From parts **a** and **b**, we know that there is no difference in the mean zinc measurement between text-line and intersection and that there is no difference in the mean zinc measurement between witness-line and intersection. However, we did not compare the mean zinc measurement between text-line and witness-line which have sample means that are the furthest apart. Thus, we can make no conclusion about the difference between these two means.

d. In order for the above inferences to be valid, we must assume:

1. The three samples are randomly selected in an independent manner from the three target populations.
2. All three sampled populations have distributions that are approximately normal.
3. All three population variances are equal (i.e. $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$)

Since there are so few observations per treatment, it is hard to check the assumptions.

9.24 Let μ_1 = mean number of high frequency vocal responses for piglets castrated using Method 1 and μ_2 = mean number of high frequency vocal responses for piglets castrated using Method 2.

Some preliminary calculations are:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(24-1).09^2 + (25-1).09^2}{24+25-2} = .0081$$

To determine if the mean number of high frequency vocal responses differ for piglets castrated by the 2 methods, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{.74 - .70}{\sqrt{.0081\left(\dfrac{1}{24} + \dfrac{1}{25}\right)}} = 1.56$

The rejection region requires $\alpha / 2 = .05/2 = .025$ in each tail of the t distribution. From Table VI, Appendix A, with df = $n_1 + n_2 - 2 = 24 + 25 - 2 = 47$, $t_{.025} \approx 2.021$. The rejection region is $t < -2.021$ or $t > 2.021$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.56 \ngtr 2.021$), H_0 is not rejected. There is insufficient evidence to indicate that the mean number of high frequency vocal responses differ for piglets castrated by the 2 methods at $\alpha = .05$.

9.26 Let μ_1 = mean performance level for students in the control group and μ_2 = mean performance level for students in the rudeness group. To determine if the mean performance level for the students in the rudeness group is lower than that for students in the control group, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 > 0$

From the printout, the test statistic is $t = 2.683$ and the p-value is $p = .0043$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the mean performance level for the students in the rudeness group is lower than that for students in the control group for any value of α greater than .0043.

9.28 Let μ_1 = mean milk price in the "surrounding" market and μ_2 = mean milk price in the Tri-county market. If the dairies participated in collusive practices, then the mean price of milk in the Tri-county market will be greater than that in the "surrounding: market. To determine if there is support for the claim of collusive practices, we test:

H_o: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The test statistic is $z = -6.02$ (from the printout).

The p-value for the test is p-value = 0.000. Since the p-value is so small, H_0 will be rejected for any reasonable value of α. There is sufficient evidence to indicate that the mean milk price in the Tri-county market is larger than that in the surrounding market. There is evidence that the dairies participated in collusive practices.

9.30 By using a paired difference experiment, one can remove sources of variation that tend to inflate the variance, σ^2. By reducing the variance, it is easier to find differences in population means that really exist.

9.32 The conditions required for a valid large-sample inference about μ_d are:

1. A random sample of differences is selected from the target population differences.
2. The sample size n_d is large, i.e., $n_d \geq 30$. (Due to the Central Limit Theorem, this condition guarantees that the test statistic will be approximately normal regardless of the shape of the underlying probability distribution of the population.)

The conditions required for a valid small-sample inference about μ_d are:

1. A random sample of differences is selected from the target population differences.
2. The population of differences has a distribution that is approximately normal.

9.34 a. H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The rejection region requires $\alpha = .10$ in the lower tail of the t distribution with df = $n_d - 1 = 16 - 1 = 15$. From Table VI, Appendix A, $t_{.10} = 1.341$. The rejection region is $t < -1.341$.

b. H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{\dfrac{s_d}{\sqrt{n_d}}} = \dfrac{-7 - 0}{\dfrac{\sqrt{64}}{\sqrt{16}}} = -3.5$

The rejection region is $t < -1.341$. (Refer to part **a**).

Since the observed value of the test statistic falls in the rejection region ($t = -3.5 < -1.341$), H_0 is rejected. There is sufficient evidence to indicate $\mu_1 - \mu_2 < 0$ at $\alpha = .10$.

c. The necessary assumptions are:

1. The population of differences is normal.
2. The differences are randomly selected.

d. For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table VI, Appendix A, with df = 15, $t_{.05} = 1.753$. The confidence interval is:

$$\bar{x}_d \pm t_{.05} \frac{s_d}{\sqrt{n_d}} \Rightarrow -7 \pm 1.753 \frac{\sqrt{64}}{\sqrt{16}} \Rightarrow -7 \pm 3.506 \Rightarrow (-10.506, -3.494)$$

We are 90% confident the mean difference is between −10.506 and −3.494.

e. The confidence interval provides more information since it gives an interval of possible values for the difference between the population means.

9.36 a. Let μ_1 = mean of population 1 and μ_2 = mean of population 2.

To determine if the mean of the population 2 is larger than the mean for population 1, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d < 0$ where $\mu_d = \mu_1 - \mu_2$

b. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Pop1, Pop2, Diff

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Pop1	10	38.40	15.06	17.00	23.50	40.50	51.25	59.00
Pop2	10	42.10	15.81	20.00	26.25	43.50	55.00	66.00
Diff	10	-3.700	2.214	-7.000	-5.250	-4.000	-1.750	0.00

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{-3.7 - 0}{2.214 / \sqrt{10}} = -5.285$

The rejection region requires $\alpha = .10$ in the lower tail of the t distribution with df = $n_d - 1$ = 10 – 1 = 9. From Table VI, Appendix A, $t_{.10} = 1.383$. The rejection region is $t < -1.383$.

Since the observed value of the test statistic falls in the rejection region ($t = -5.285 < -1.383$), H_0 is rejected. There is sufficient evidence to indicate the mean for population 1 is less than the mean for population 2 at $\alpha = .10$.

c. For confidence coefficient .90, $\alpha = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table VI, Appendix A, with df = $n_d - 1 = 10 - 1 = 9$, $t_{.05} = 1.833$. The 90% confidence interval is:

$$\bar{x}_d \pm t_{.05} \frac{s_d}{\sqrt{n_d}} \Rightarrow -3.7 \pm 1.833 \frac{2.214}{\sqrt{10}} \Rightarrow -3.7 \pm 1.28 \Rightarrow (-4.98, -2.42)$$

We are 90% confident that the difference in the means of the 2 populations is between –4.98 and –2.42.

d. The assumptions necessary are that the distribution of the population of differences is normal and that the sample is randomly selected.

9.38 a. Let μ_1 = mean BMI at the start of camp and μ_2 = mean BMI at the end of camp. The parameter of interest is $\mu_d = \mu_1 - \mu_2$. To determine if the mean BMI at the end of camp is less than the mean BMI at the start of camp, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d > 0$

b. It should be analyzed as a paired-difference t-test. The BMI was measured on each adolescent at the beginning and at the end of camp. The samples are not independent.

c. The test statistics is $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} = \dfrac{34.9 - 31.6}{\sqrt{\dfrac{6.9^2}{76} + \dfrac{6.2^2}{76}}} = 3.10$

d. The test statistic is $z = \dfrac{\bar{x}_d - 0}{\sqrt{\dfrac{s_d^2}{n_d}}} = \dfrac{3.3}{\sqrt{\dfrac{1.5^2}{76}}} = 19.18$

e. The test statistic in part c is much smaller than the test statistic in part d. The test statistic in part d provides more evidence in support of the alternative hypothesis.

f. Since the p-value is smaller than α ($p < .0001 < .01$), H_0 is rejected. There is sufficient evidence to indicate the mean BMI at the end of camp is less than the mean BMI at the beginning of camp at $\alpha = .01$.

g. No. Since the sample size is sufficiently large ($n_d = 76$), the Central Limit Theorem applies.

h. For confidence coefficient .99, $\alpha = .01$ and $\alpha / 2 = .005$. From Table IV, Appendix A, $z_{.005} = 2.58$. The confidence interval is:

$$\bar{x}_d \pm z_{.005}\sqrt{\frac{s_d^2}{n_d}} \Rightarrow 3.3 \pm 2.58\sqrt{\frac{1.5^2}{76}} \Rightarrow 3.3 \pm .44 \Rightarrow (2.86,\ 3.74)$$

We are 99% confident that the true difference in mean BMI between the beginning and end of camp is between 2.86 and 3.74.

9.40 a. The data should be analyzed as a paired difference experiment because there were 2 measurements on each person or experimental unit, the number of laugh episodes as a speaker and the number of laugh episodes as an audience member. These 2 observations on each person are not independent of each other.

b. The study's target parameter is μ_d = difference in the mean number of laugh episodes between the speaker and the audience member.

c. No. With just the sample means of the speakers and audience, we do not have enough information to make a decision. We also need to know the variance of the differences.

d. With a p-value of $p < .01$, we would reject H_o for any value of $\alpha \geq .01$. There is sufficient evidence to indicate a difference in the mean number of laugh episodes between speakers and audience members at $\alpha \geq .01$

9.42 a. Let μ_1 = mean severity of the driver's chest injury and μ_2 = mean severity of the passenger's chest injury. The target parameter is $\mu_d = \mu_1 - \mu_2$, the difference in the mean severity of the driver's and passenger's chest injuries.

b. For each car, there are measures for the severity of the driver's chest injury and the severity of the passenger's chest injury. Since both measurements came from the same car, they are not independent.

c. Using MINITAB, the descriptive statistics for the difference data (driver chest – passenger chest) are:

Descriptive Statistics: Crash-Diff

```
Variable     N    Mean  StDev  Minimum      Q1 Median      Q3   Maximum
Crash-Diff 98 -0.561  5.517  -15.000 -4.000  0.000  3.000    13.000
```

For confidence coefficient .99, $\alpha = .01$ and $\alpha / 2 = .01/2 = .005$. From Table IV, appendix A, $z_{.005} = 2.58$. The 99% confidence interval is:

$$\bar{x}_d \pm z_{.005} \frac{s_d}{\sqrt{n_d}} \Rightarrow -.561 \pm 2.58 \frac{5.517}{\sqrt{98}} \Rightarrow -.561 \pm 1.438 \Rightarrow (-1.999,\ \ 0.877)$$

d. We are 99% confident that the difference in the mean severity of chest injury between drivers and passengers is between −1.999 and 0.877. Since 0 is contained in this interval, there is no evidence of a difference in the mean severity of chest injuries between drivers and passengers at $\alpha = .01$.

e. Since the sample size is so large (n = number of pairs = 98), the Central Limit Theorem applies. Thus, the only necessary condition is that the sample is random from the target population of differences.

9.44 a. The data should be analyzed using a paired-difference analysis because that is how the data were collected. Reaction times were collected twice from each subject, once under the random condition and once under the static condition. Since the two sets of data are not independent, they cannot be analyzed using independent samples analyses.

b. Let μ_1 = mean reaction time under the random condition and μ_2 = mean reaction time under the static condition. Let $\mu_d = \mu_1 - \mu_2$. To determine if there is a difference in mean reaction time between the two conditions, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

c. The test statistic is $t = 1.52$ with a p-value of .15. Since the p-value is not small, there is no evidence to reject H_0 for any reasonable value of α. There is insufficient evidence to indicate a difference in the mean reaction times between the two conditions. This supports the researchers' claim that visual search has no memory.

9.46 a. To determine if 2 population means are different, we not only need to know the difference in the sample means, but we also need to know the variance of the sample differences. If the variance is small, then we probably could conclude that the means are different. If the variance is large, then we might not be able to conclude the means are different.

b. Since the p-value is so small ($p < .001$), H_0 is rejected. There is sufficient evidence to indicate the mean score on the Quick-REST Survey at the end of the workshop is greater than the mean at the beginning of the workshop for any reasonable value of α. The program was effective.

c. We must assume that the sample is random. Since the number of pairs is large ($n = 238$), the Central Limit Theorem applies.

9.48 a. Let $\mu_1 =$ mean standardized growth of genes in the full-dark condition and $\mu_2 =$ mean standardized growth of genes in the transient light condition. Then $\mu_d = \mu_1 - \mu_2 =$ difference in mean standardized growth between genes in full-dark condition and genes in transient light condition.

Some preliminary calculations are:

Gene ID	**Full-Dark**	**Transient Light**	**Difference**
SLR2067	-0.00562	1.40989	-1.41551
SLR1986	-0.68372	1.83097	-2.51469
SLR3383	-0.25468	-0.79794	0.54326
SLR0928	-0.18712	-1.20901	1.02189
SLR0335	-0.20620	1.71404	-1.92024
SLR1459	-0.53477	2.14156	-2.67633
SLR1326	-0.06291	1.03623	-1.09914
SLR1329	-0.85178	-0.21490	-0.63688
SLR1327	0.63588	1.42608	-0.79020
SLR1325	-0.69866	1.93104	-2.62970

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{-12.11754}{10} = -1.212$$

$$s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{29.67025188 - \frac{(-12.11754)^2}{10}}{10-1} = \frac{14.98677431}{9} = 1.665197146$$

$$s = \sqrt{1.665197146} = 1.290$$

To determine if there is a difference in mean standardized growth of genes in the full-dark condition and genes in the transient light condition, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{-1.212 - 0}{1.290 / \sqrt{10}} = -2.97$.

The rejection region requires $\alpha / 2 = .01 / 2 = .005$ in each tail of the t distribution with df = $n_d - 1 = 10 - 1 = 9$. From Table VI, Appendix A, $t_{.005} = 3.250$. The rejection region is $t < -3.250$ or $t > 3.250$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -2.97 \nless -3.250$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean standardized growth of genes in the full-dark condition and genes in the transient light condition at $\alpha = .01$.

b. Using MINITAB, the mean difference in standardized growth of the 103 genes in the full-dark condition and the transient light condition is:

Descriptive Statistics: FD-TL

```
Variable    N    Mean  StDev  Minimum      Q1  Median      Q3  Maximum
FD-TL     103  -0.420  1.422   -2.765  -1.463  -0.296  0.803    2.543
```

The population mean difference is –.420. The test in part **a** did not detect this difference.

c. Let $\mu_3 =$ mean standardized growth of genes in the transient dark condition. Then $\mu_d = \mu_1 - \mu_3 =$ difference in mean standardized growth between genes in full-dark condition and genes in transient dark condition.

Some preliminary calculations are:

Gene ID	Full-Dark	Transient Dark	Difference
SLR2067	–0.00562	–1.28569	1.28007
SLR1986	–0.68372	–0.68723	0.00351
SLR3383	–0.25468	–0.39719	0.14251
SLR0928	–0.18712	–1.18618	0.99906
SLR0335	–0.20620	–0.73029	0.52409
SLR1459	–0.53477	–0.33174	-0.20303
SLR1326	–0.06291	0.30392	-0.36683
SLR1329	–0.85178	0.44545	-1.29723
SLR1327	0.63588	–0.13664	0.77252
SLR1325	–0.69866	–0.24820	-0.45046

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{1.40421}{10} = 0.140$$

$$s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{5.589984302 - \frac{(1.40421)^2}{10}}{10 - 1} = \frac{5.39280373}{9} = 0.599200414$$

$$s = \sqrt{0.599200414} = .774$$

To determine if there is a difference in mean standardized growth of genes in the full-dark condition and genes in the transient dark condition, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{.140 - 0}{.774 / \sqrt{10}} = 0.57$.

From part **a**, the rejection region is $t < -3.250$ or $t > 3.250$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 0.57 \not> 3.250$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean standardized growth of genes in the full-dark condition and genes in the transient dark condition at $\alpha = .01$.

Using MINITAB, the mean difference in standardized growth of the 103 genes in the full dark condition and the transient dark condition is:

Descriptive Statistics: FD-TD

```
Variable    N     Mean   StDev  Minimum       Q1   Median       Q3  Maximum
FD-TD     103 -0.2274  0.9239  -2.8516  -0.8544  -0.1704   0.2644   2.3998
```

The population mean difference is $-.2274$. The test above did not detect this difference.

d. $\mu_d = \mu_2 - \mu_3$ = difference in mean standardized growth between genes in transient light condition and genes in transient dark condition.

Some preliminary calculations are:

Gene ID	Transient Light	Transient Dark	Difference
SLR2067	1.40989	–1.28569	2.69558
SLR1986	1.83097	–0.68723	2.51820
SLR3383	–0.79794	–0.39719	–0.40075
SLR0928	–1.20901	–1.18618	–0.02283
SLR0335	1.71404	–0.73029	2.44433
SLR1459	2.14156	–0.33174	2.47330
SLR1326	1.03623	0.30392	0.73231
SLR1329	–0.21490	0.44545	–0.66035
SLR1327	1.42608	–0.13664	1.56272
SLR1325	1.93104	–0.24820	2.17924

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{13.52175}{10} = 1.352$$

$$s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{34.02408743 - \frac{(13.52175)^2}{10}}{10-1} = \frac{15.74031512}{9} = 1.748923902$$

$$s = \sqrt{1.748923902} = 1.322$$

To determine if there is a difference in mean standardized growth of genes in the transient light condition and genes in the transient dark condition, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \frac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \frac{1.352 - 0}{1.322 / \sqrt{10}} = 3.23$.

From part **a**, the rejection region is t < -3.250 or t > 3.250.

Since the observed value of the test statistic does not fall in the rejection region ($t = 3.23 \ngtr 3.250$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean standardized growth of genes in the transient light condition and genes in the transient dark condition at $\alpha = .01$.

Using MINITAB, the mean difference in standardized growth of the 103 genes in the full-dark condition and the transient dark condition is:

Descriptive Statistics: TL-TD

```
Variable   N  Mean  StDev  Minimum     Q1  Median     Q3  Maximum
TL-TD    103 0.192  1.499   -3.036 -1.166   0.149  1.164    2.799
```

The population mean difference is .192. The test above did not detect this difference.

9.50 Let μ_1 = mean density of the wine measured with the hydrometer and μ_2 = mean density of the wine measured with the hydrostatic balance. The target parameter is $\mu_d = \mu_1 - \mu_2$, the difference in the mean density of the wine measured with the hydrometer and the hydrostatic balance.

Using MINITAB, the descriptive statistics for the difference data are:

Descriptive Statistics: Diff

```
Var    N      Mean    StDev   Minimum        Q1    Median        Q3  Maximum
Diff  40 -0.000523 0.001291 -0.004480 -0.001078 -0.000165 0.000317 0.001580
```

We will use a 95% confidence interval to estimate the difference in the mean density of the wine measured with the hydrometer and the hydrostatic balance. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05/2 = .025$. From Table IV, appendix A, $z_{.025} = 1.96$. The 95% confidence interval is:

$$\bar{x}_d \pm z_{.025}\frac{s_d}{\sqrt{n_d}} \Rightarrow -.000523 \pm 1.96\frac{.001292}{\sqrt{40}} \Rightarrow -.000523 \pm .000400 \Rightarrow (-.000923,\ \ -.000123)$$

We are 95% confident that the difference in the mean density of the wine measured with the hydrometer and the hydrostatic balance is between −.000923 and −.000123. Thus, we are 95% confident that the difference in the means ranges from .000123 and .000923 in absolute value. Since this entire confidence interval is less than .002, we can conclude that the difference in the mean scores does not exceed .002. Thus, we would recommend that the winery switch to the hydrostatic balance.

9.52 If both sample sizes are small, then the sampling distribution of $(\hat{p}_1 - \hat{p}_2)$ may not be normal.

9.54 The sample sizes are large enough if $n_1\hat{p}_1 \geq 15$, $n_1\hat{q}_1 \geq 15$ and $n_2\hat{p}_2 \geq 15$, $n_2\hat{q}_2 \geq 15$.

a. $n_1\hat{p}_1 = 10(.5) = 5 \ngeq 15, \quad n_1\hat{q}_1 = 10(.5) = 5 \ngeq 15$

$n_2\hat{p}_2 = 12(.5) = 6 \ngeq 15, \quad n_2\hat{q}_2 = 12(.5) = 6 \ngeq 15$

Since none of the products are greater than or equal to 15, the sample sizes are not large enough to assume normality.

b. $n_1\hat{p}_1 = 10(.1) = 1 \ngeq 15, \quad n_1\hat{q}_1 = 10(.9) = 9 \ngeq 15$

$n_2\hat{p}_2 = 12(.08) = .96 \ngeq 15, \quad n_2\hat{q}_2 = 12(.92) = 11.04 \ngeq 15$

Since none of the products are greater than or equal to 15, the sample sizes are not large enough to assume normality.

c. $n_1\hat{p}_1 = 30(.2) = 6 \ngeq 15, \quad n_1\hat{q}_1 = 30(.8) = 24 > 15$

$n_2\hat{p}_2 = 30(.3) = 9 \ngeq 15, \quad n_2\hat{q}_2 = 30(.7) = 21 > 15$

Since two of the products are not greater than or equal to 15, the sample sizes are not large enough to assume normality.

d. $n_1\hat{p}_1 = 100(.05) = 5 \ngeq 15, \quad n_1\hat{q}_1 = 100(.95) = 95 > 15$

$n_2\hat{p}_2 = 200(.09) = 18 > 15, \quad n_2\hat{q}_2 = 200(.91) = 182 > 15$

Since one of the products is not greater than or equal to 15, the sample sizes are not large enough to assume normality.

e. $n_1\hat{p}_1 = 100(.95) = 95 > 15, \quad n_1\hat{q}_1 = 100(.05) = 5 \ngeq 15$

$n_2\hat{p}_2 = 200(.91) = 182 > 15, \quad n_2\hat{q}_2 = 200(.09) = 18 > 15$

Since one of the products is not greater than or equal to 15, the sample sizes are not large enough to assume normality.

9.56 a. H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 \neq 0$

Will need to calculate the following:

$$\hat{p}_1 = \frac{320}{800} = .40 \qquad \hat{p}_2 = \frac{40}{800} = .50 \qquad \hat{p} = \frac{320 + 400}{800 + 800} = .45$$

The test statistic is $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{(.40 - .50) - 0}{\sqrt{(.45)(.55)\left(\dfrac{1}{800} + \dfrac{1}{800}\right)}} = -4.02$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic falls in the rejection region ($z = -4.02 < -1.96$), H_0 is rejected. There is sufficient evidence to indicate that the proportions are unequal at $\alpha = .05$.

b. The problem is identical to part **a** until the rejection region. The rejection region requires $\alpha / 2 = .01 / 2 = .005$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.005} = 2.58$. The rejection region is $z < -2.58$ or $z > 2.58$.

Since the observed value of the test statistic falls in the rejection region ($z = -4.02 < -2.58$), H_0 is rejected. There is sufficient evidence to indicate that the proportions are unequal at $\alpha = .01$.

c. H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 < 0$

Test statistic as above: $z = -4.02$

The rejection region requires $\alpha = .01$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.01} = 2.33$. The rejection region is $z < -2.33$.

Since the observed value of the test statistic falls in the rejection region ($z = -4.02 < -2.33$), H_0 is rejected. There is sufficient evidence to indicate that $p_1 < p_2$ at $\alpha = .01$.

d. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, appendix A, $z_{.05} = 1.645$. The 90% confidence interval is:

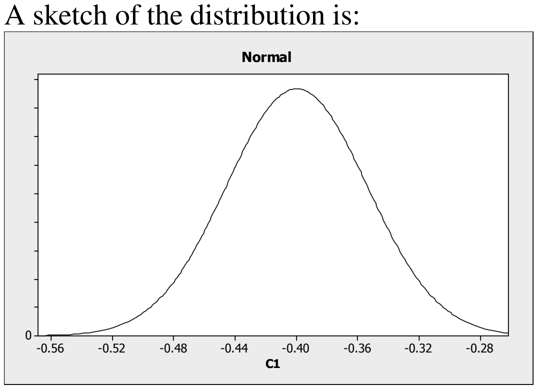
$$(\hat{p}_1 - \hat{p}_2) \pm z_{.05}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \Rightarrow (.4 - .5) \pm 1.645\sqrt{\frac{(.4)(.6)}{800} + \frac{(.5)(.5)}{800}}$$
$$\Rightarrow -.10 \pm .04 \Rightarrow (-.14, -.06)$$

9.58 The sampling distribution $\hat{p}_1 - \hat{p}_2$ is approximately normal with:

$$\mu_{(\hat{p}_1 - \hat{p}_2)} = p_1 - p_2 = .1 - .5 = -.4$$

$$\sigma_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} = \sqrt{\frac{(.1)(.9)}{100} + \frac{(.5)(.5)}{200}} = .046$$

A sketch of the distribution is:



9.60 a. Let p_1 = proportion of all Democrats who prefer steak as their favorite barbeque food. The point estimate of p_1 is $\hat{p}_1 = \frac{662}{1,250} = .530$.

b. Let p_2 = proportion of all Republicans who prefer steak as their favorite barbeque food. The point estimate of p_2 is $\hat{p}_2 = \frac{586}{930} = .630$.

c. The point estimate of the difference between the proportions of all Democrats and all Republicans who prefer steak as their favorite barbeque food is $\hat{p}_1 - \hat{p}_2 = .530 - .630 = -.100$.

d. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The 95% confidence interval is:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \Rightarrow (.530 - .630) \pm 1.96\sqrt{\frac{.53(.47)}{1250} + \frac{.63(.37)}{930}}$$
$$\Rightarrow -.100 \pm .042 \Rightarrow (-.142,\ -.058)$$

e. We are 95% confident that the difference between the proportions of all Democrats and all Republicans who prefer steak as their favorite barbeque food is between -.142 and -.058. Thus, there is evidence that Republicans prefer steak as their favorite barbeque food more than Democrats.

f. 95% confidence means that if we were to take repeated samples of size 1,250 Democrats and 930 Republicans and form 95% confident intervals for the difference in the population parameters, 95% of the confidence intervals formed will contain the true difference and 5% will not.

9.62 a. Let p_1 = proportion of men who prefer keeping track of appointments in their heads and p_2 = proportion of women who prefer keeping track of appointments in their heads. To determine whether the percentage of men who prefer keeping track of appointments in their heads is larger than the corresponding percentage of women, we test:

H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 > 0$

b. Some preliminary calculations:

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_1\hat{p}_1}{n_1 + n_2} = \frac{500(.56) + 500(.46)}{500 + 500} = .51$$

The test statistic is $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{(.56 - .46) - 0}{\sqrt{.51(.49)\left(\dfrac{1}{500} + \dfrac{1}{500}\right)}} = 3.16$

c. The rejection region requires $\alpha = .01$ in the upper tail of the z distribution. From Table IV, Appendix A, $z_{.01} = 2.33$. The rejection region is $z > 2.33$.

d. The p-value is $p = P(z \geq 3.16) \approx .5 - .5 = 0$.

e. Since the observed value of the test statistic falls in the rejection region ($z = 3.16 > 2.33$), H_0 is rejected. There is sufficient evidence to indicate the percentage of men who prefer keeping track of appointments in their heads is larger than the corresponding percentage of women at $\alpha = .01$.

OR

Since the p-value is less than α ($p = 0 < \alpha = .01$), H_0 is rejected. There is sufficient evidence to indicate the percentage of men who prefer keeping track of appointments in their heads is larger than the corresponding percentage of women at $\alpha = .01$.

9.64 Let p_1 = proportion of patients in the angioplasty group who had a heart attack and p_2 = proportion of patients in the medication only group who had a heart attack. The parameter of interest is $p_1 - p_2$, or the difference in the proportions of patients who had a heart attack between the 2 groups.

Some preliminary calculations are:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{211}{1{,}145} = .184 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{202}{1{,}142} = .177$$

For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table IV, Appendix A, $z_{.025}$ = 1.96. The 95% confidence interval is:

$$(.184 - .177) \pm 1.96\sqrt{\frac{.184(.816)}{1{,}145} + \frac{.177(.823)}{1{,}142}} \Rightarrow .007 \pm .032 \Rightarrow (-.025, \;\; .039)$$

We are 95% confident that the difference in the proportions of patients who had a heart attack between the 2 groups is between −.025 and .039. Since 0 is contained in the interval, there is no evidence that there is a difference in the proportion of heart attacks between those who received angioplasty and those who received medication only at $\alpha = .05$.

9.66 Let p_1 = proportion of world-class athletes not competing in the 1999 World Championships and p_2 = proportion of world-class athletes not competing in the 2000 Olympic Games. If the new EPO test is effective, then the proportion of nonparticipating athletes will increase.

Some preliminary calculations are:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{159}{830} = .1916 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{133}{825} = .1612 \qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{159 + 133}{833 + 825} = .1764$$

To determine if the new test for EPO was effective in deterring an athlete's participation in the 2000 Olympics, we test:

H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 < 0$

The test statistic is $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{.1916 - .1612}{\sqrt{.1764(.8236)\left(\dfrac{1}{830} + \dfrac{1}{825}\right)}} = 1.62$

The rejection region requires $\alpha = .10$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.10}$ = 1.28. The rejection region is $z < -1.28$.

Since the observed value of the test statistic does not fall in the rejection region ($z = 1.62 \not< -1.28$), H_0 is not rejected. There is insufficient evidence to indicate the new test was effective in deterring an athlete's participation in the 2000 Olympics at $\alpha = .10$.

9.68 a. The estimate of the incumbency rate in the surrounding milk market is:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{91}{134} = .679$$

The estimate of the incumbency rate in the Tri-county milk market is:

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{50}{51} = .980$$

b. To determine if the incumbency rate for the Tri-county milk market is greater than that of the surrounding milk market, we test:

H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 < 0$

From the printout, the test statistic is $z = -4.30$.

The p-value for the test is p-value = 0.000. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the incumbency rate for the Tri-county milk market is greater than that of the surrounding milk market for any reasonable value of α. This is evidence that bid collusion was present.

9.70 Let p_1 = proportion of all commercials in 1998 that contain religious symbolism and p_2 = proportion of all commercials in 2008 that contain religious symbolism. Some preliminary calculations are:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{16}{797} = .020 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{51}{1,499} = .034 \qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{16+51}{797+1,499} = .029$$

To determine if the percentage of commercials that use religious symbolism has changes since the 1998 study, we test:

H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 \neq 0$

The test statistic is $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{.020 - .034}{\sqrt{.029(.971)\left(\dfrac{1}{797} + \dfrac{1}{1,499}\right)}} = -1.90$

The p-value of the test is $P(z \le -1.90) + P(z \ge 1.90) = 2(.5 - .4713) = 2(.0287) = .0574$.

Since no α was given, we will use $\alpha = .05$. Since the p-value is larger than α (p = .0574 > .05), H_0 is not rejected. There is insufficient evidence to indicate the percentage of commercials that use religious symbolism has changes since the 1998 study at $\alpha = .05$.

9.72 We can obtain estimates of σ_1^2 and σ_2^2 by using the sample variances s_1^2 and s_2^2, or from an educated guess based on the range – s ≈ Range / 4.

9.74 If the sample size calculation yields a value of n that is too large to be practical, we might decide to use a larger sampling error (SE) in order to reduce the sample size, or we might decrease the confidence coefficient.

9.76 a. For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table IV, Appendix A, $z_{.025}$ = 1.96.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(1.96)^2(15^2 + 17^2)}{3.2^2} = 192.83 \approx 193$$

b. If the range of each population is 40, we would estimate σ by:

$\sigma \approx 60 / 4 = 15$

For confidence coefficient .99, $\alpha = 1 - .99 = .01$ and $\alpha / 2 = .01 / 2 = .005$. From Table IV, Appendix A, $z_{.005}$ = 2.58.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(2.58)^2(15^2 + 15^2)}{8^2} = 46.8 \approx 47$$

c. For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table IV, Appendix A, $z_{.05}$ = 1.645. For a width of 1, the standard error is .5.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(1.645)^2(5.8 + 7.5)}{.5^2} = 143.96 \approx 144$$

9.78 For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table IV, Appendix A, $z_{.05}$ = 1.645. For width = .1, the standard error is SE = .1/2 = .05.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2(p_1q_1 + p_2q_2)}{(SE)^2} = \frac{1.645^2(.6(.4) + .6(.4))}{.05^2} = 519.6 \approx 520$$

In order to estimate the difference in proportions using a 90% confidence interval of width .1, we would need to sample 520 observations from each population. Since only enough money was budgeted to sample 100 observations from each population, insufficient funds have been allocated.

9.80 To determine the number of pairs to use, we can use the formula for a single sample. For confidence coefficient .90, $\alpha = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table IV, Appendix A, $z_{.05}$ = 1.645.

$$n_d = \frac{\left(z_{\alpha/2}\right)^2\sigma_d^2}{\left(SE\right)^2} = \frac{1.645^2(3)}{\left(.75\right)^2} = 14.4 \approx 15$$

9.82 a. For confidence coefficient .80, $\alpha = 1 - .80 = .20$ and $\alpha / 2 = .20 / 2 = .10$. From Table IV, Appendix A, $z_{.10}$ = 1.28. Since we have no prior information about the proportions, we use $p_1 = p_2$ = .5 to get a conservative estimate. For a width of .06, the standard error is .03.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2(p_1q_1 + p_2q_2)}{(SE)^2} = \frac{(1.28)^2(.5(1 - .5) + .5(.1 - .5))}{.03^2} = 910.22 \approx 911$$

b. For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. Using the formula for the sample size needed to estimate a proportion from Chapter 7,

$$n = \frac{(z_{\alpha/2})^2 pq}{(SE)^2} = \frac{1.645^2(.5(1-.5))}{.02^2} = \frac{.6765}{.0004} = 1691.27 \approx 1692$$

No, the sample size from part **a** is not large enough.

9.84 For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. Since no prior information was given as to the values of the p's, we will use $p = .5$ for both populations.

$$n_1 = n_2 = \frac{(z_{\alpha/2})^2(p_1q_1 + p_2q_2)}{(SE)^2} = \frac{1.96^2(.5(.5)+.5(.5))}{.02^2} = 4{,}802$$

In order to estimate the difference in proportions to within .02 using a 90% confidence interval, we would need to sample 4,802 observations from each population.

9.86 The sampling distribution of s_1^2 / s_2^2 for normal data is the F distribution with $n_1 - 1$ numerator degrees of freedom and $n_2 - 1$ denominator degrees of freedom.

9.88 The statement "The F statistic used for testing H_0: $\sigma_1^2 = \sigma_2^2$ against H_a: $\sigma_1^2 < \sigma_2^2$ is $F = (s_1)^2 / (s_2)^2$ " is false. The test statistic should be $F = (s_2)^2 / (s_1)^2$. If we are trying to show that $\sigma_2^2 > \sigma_1^2$, then $s_2^2 > s_1^2$. The F statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smallersample variance}}$.

9.90 a. $F_{.05} = 4.82$ where $\nu_1 = 8$ and $\nu_2 = 5$ from Table IX.

b. $F_{.01} = 3.51$ where $\nu_1 = 20$ and $\nu_2 = 14$ from Table XI.

c. $F_{.025} = 6.62$ where $\nu_1 = 10$ and $\begin{matrix}\nu_1 = 20 \\ \nu_2 = 30\end{matrix}$ from Table X.

d. $F_{.10} = 3.21$ where $\nu_1 = 20$ and $\nu_2 = 5$ from Table VIII.

9.92 a. The rejection region requires $\alpha = .10$ in the upper tail of the F distribution with $\nu_1 = 20$ and $\nu_2 = 30$. From Table VIII, Appendix A, $F_{.10} = 1.67$. The rejection region is $F > 1.67$.

b. The rejection region requires $\alpha = .05$ in the upper tail of the F distribution with $\nu_1 = 20$ and $\nu_2 = 30$. From Table IX, Appendix A, $F_{.05} = 1.93$. The rejection region is $F > 1.93$.

c. The rejection region requires $\alpha = .025$ in the upper tail of the F distribution with $\nu_1 = 20$ and $\nu_2 = 30$. From Table X, Appendix A, $F_{.025} = 2.20$. The rejection region is $F > 2.20$.

d. The rejection region requires $\alpha = .01$ in the upper tail of the F distribution with $\nu_1 = 20$ and $\nu_2 = 30$. From Table XI, Appendix A, $F_{.01}$ = 2.55. The rejection region is $F > 2.55$.

9.94 a. The rejection region requires $\alpha = .05$ in the upper tail of the F distribution with $\nu_1 = n_1 - 1 = 25 - 1 = 24$ and $\nu_2 = n_2 - 1 = 20 - 1 = 19$. From Table IX, Appendix A, $F_{.05}$ = 2.11. The rejection region is $F > 2.11$ (if $s_1^2 > s_2^2$).

b. The rejection region requires $\alpha = .05$ in the upper tail of the F distribution with $\nu_1 = n_2 - 1 = 15 - 1 = 14$ and $\nu_2 = n_1 - 1 = 10 - 1 = 9$. From Table IX, Appendix A, $F_{.05} \approx 3.01$. The rejection region is $F > 3.01$ (if $s_2^2 > s_1^2$).

c. The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in the upper tail of the F distribution. If $s_1^2 > s_2^2$, $\nu_1 = n_1 - 1 = 21 - 1 = 20$ and $\nu_2 = n_2 - 1 = 31 - 1 = 30$. From Table IX, Appendix A, $F_{.05}$ = 1.93. The rejection region is $F > 1.93$. If $s_1^2 < s_2^2$, $\nu_1 = n_2 - 1 = 30$ and $\nu_2 = n_1 - 1 = 20$. From Table IX, $F_{.05}$ = 2.04. The rejection region is $F > 2.04$.

d. The rejection region requires $\alpha = .01$ in the upper tail of the F distribution with $\nu_1 = n_2 - 1 = 41 - 1 = 40$ and $\nu_2 = n_1 - 1 = 31 - 1 = 30$. From Table XI, Appendix A, $F_{.01}$ = 2.30. The rejection region is $F > 2.30$ (if $s_2^2 > s_1^2$).

e. The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in the upper tail of the F distribution. If $s_1^2 > s_2^2$, $\nu_1 = n_1 - 1 = 7 - 1 = 6$ and $\nu_2 = n_2 - 1 = 16 - 1 = 15$. From Table X, Appendix A, $F_{.025}$ = 3.14. The rejection region is $F > 3.14$. If $s_1^2 < s_2^2$, $\nu_1 = n_2 - 1 = 15$ and $\nu_2 = n_1 - 1 = 6$. From Table X, Appendix A, $F_{.025}$ = 5.27. The rejection region is $F >$ 5.27.

9.96 We need to calculate the sample variances.

$$s_1^2 = \frac{\sum x_1^2 - \frac{\left(\sum x_1\right)^2}{n_1}}{n_1 - 1} = \frac{\left(44.35 - \frac{(14.3)^2}{6}\right)}{6-1} = 2.054$$

$$s_2^2 = \frac{\sum x_2^2 - \frac{\left(\sum x_2\right)^2}{n_2}}{n_2 - 1} = \frac{\left(100.15 - \frac{(16.3)^2}{4}\right)}{4-1} = 11.2425$$

a. H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 < \sigma_2^2$

where σ_1^2 = the variance in population 1
σ_2^2 = the variance in population 2

The test statistic is $F = \frac{s_2^2}{s_1^2} = \frac{11.2425}{2.054} = 5.47$

The rejection region requires $\alpha = .01$ in the upper tail of the F distribution with $\nu_1 = n_2 - 1 = 4 - 1 = 3$ and $\nu_2 = n_1 - 1 = 6 - 1 = 5$. From Table XI, Appendix A, $F_{.01}$ = 12.06. The rejection region is $F > 12.06$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 5.47 \ngtr 12.06$), H_0 is not rejected. There is insufficient evidence to conclude that the variances in the two populations are different at $\alpha = .01$.

b. H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

The test statistic is $F = \frac{s_2^2}{s_2^1} = \frac{11.2425}{2.054} = 5.47$

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in the upper tail of the F distribution with $\nu_1 = n_2 - 1 = 4 - 1 = 3$ and $\nu_2 = n_1 - 1 = 6 - 1 = 5$. From Table IX, Appendix A, $F_{.05}$ = 5.41. The rejection region is $F > 5.41$.

Since the observed value of the test statistic falls in the rejection region ($F = 5.47 > 5.41$), H_0 is rejected. There is sufficient evidence to conclude that the variances in the two populations are different at $\alpha = .10$.

9.98 Let σ_B^2 = variance of the FNE scores for bulimic students and σ_N^2 = variance of the FNE scores for normal students.

From Exercise 9.19, $s_B^2 = 24.1636$ and $s_N^2 = 27.9780$

To determine if the variances are equal, we test:

H_0: $\sigma_B^2 = \sigma_N^2$
H_a: $\sigma_B^2 \neq \sigma_N^2$

The test statistic is $F = \frac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \frac{s_N^2}{s_B^2} = \frac{27.9780}{24.1636} = 1.16$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in the upper tail of the F distribution with numerator df $\nu_1 = n_2 - 1 = 14 - 1 = 13$ and denominator df $\nu_2 = n_1 - 1 = 11 - 1 = 10$. From Table X, Appendix A, $F_{.025} \approx 3.62$. The rejection region is $F > 3.62$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 1.16 \ngtr 3.62$), H_0 is not rejected. There is insufficient evidence that the two variances are different at $\alpha = .05$. It appears that the assumption of equal variances is valid.

9.100 a. Let σ_1^2 = variance of the number of ant species in the Dry Steppe and σ_2^2 = variance of the number of ant species in the Gobi Dessert.

To determine if the two variances are the same, we test:

H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

b. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Species

Variable	Region	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Species	Dry Steppe	5	14.00	21.31	3.00	3.00	5.00	29.50	52.00
	Gobi Desert	6	11.83	18.21	4.00	4.00	4.50	16.00	49.00

The test statistic is $F = \frac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \frac{s_1^2}{s_2^2} = \frac{21.31^2}{18.21^2} = 1.369$

c. The rejection region requires $\alpha/2 = .05/2 = .025$ in the upper tail of the F distribution with $\nu_1 = n_1 - 1 = 5 - 1 = 4$ and $\nu_2 = n_2 - 1 = 6 - 1 = 5$. Using Table X, Appendix A, $F_{.025}$ = 7.39. The rejection region is $F > 7.39$.

d. The p-value is $p = 2P(F \geq 1.369)$. Using Table VIII with $\nu_1 = n_1 - 1 = 5 - 1 = 4$ and $\nu_2 = n_2 - 1 = 6 - 1 = 5$, $P(F > 3.52) = .10$. Thus, the p-value is:

$p = 2P(F \geq 1.369) > 2(.10) = .20.$

e. Since the observed value of the test statistic does not fall in the rejection region (F = 1.369 $\ngtr$ 7.39), H_0 is not rejected. There is insufficient evidence to indicate the two variances are different at $\alpha = .05$.

f. The conditions necessary for the test results to be valid are:

1. The two sampled populations are normally distributed.
2. The samples are randomly and independently selected from their respective populations.

9.102 a. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Text-line, Witness-line, Intersection

Variable	N	Mean	Median	StDev	Minimum	Maximum	Q1	Q3
Text-lin	3	0.3830	0.3740	0.0531	0.3350	0.4400	0.3350	0.4400
Witness-	6	0.3042	0.2955	0.1015	0.1880	0.4390	0.2045	0.4075
Intersec	5	0.3290	0.3190	0.0443	0.2850	0.3930	0.2900	0.3730

Let σ_1^2 = variance of zinc measurements for the text-line, σ_2^2 = variance of zinc measurements for the witness-line, and σ_3^2 = variance of zinc measurements for the intersection.

To determine if the variation in zinc measurements for the text-line and the variation in zinc measurements for the intersection differ, we test:

H_0: $\sigma_1^2 = \sigma_3^2$
H_a: $\sigma_1^2 \neq \sigma_3^2$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_1^2}{s_3^2} = \dfrac{.0531^2}{.0443^2} = 1.437$

The rejection region requires $\alpha/2 = .05/2 = .025$ in the upper tail of the F distribution with $\nu_1 = n_1 - 1 = 3 - 1 = 2$ and $\nu_2 = n_3 - 1 = 5 - 1 = 4$. Using Table X, Appendix A, $F_{.025} = 10.65$. The rejection region is $F > 10.65$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 1.437 \not> 10.65$), H_0 is not rejected. There is insufficient evidence to indicate the variation in zinc measurements for the text-line and the variation in zinc measurements for the intersection differ at $\alpha = .05$.

b. To determine if the variation in zinc measurements for the witness-line and the variation in zinc measurements for the intersection differ, we test:

H_0: $\sigma_2^2 = \sigma_3^2$
H_a: $\sigma_2^2 \neq \sigma_3^2$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_2^2}{s_3^2} = \dfrac{.1015^2}{.0443^2} = 5.250$

The rejection region requires $\alpha/2 = .05/2 = .025$ in the upper tail of the F distribution with $\nu_1 = n_2 - 1 = 6 - 1 = 5$ and $\nu_2 = n_3 - 1 = 5 - 1 = 4$. Using Table X, Appendix A, $F_{.025} = 9.36$. The rejection region is $F > 9.36$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 5.250 \not> 9.36$), H_0 is not rejected. There is insufficient evidence to indicate the variation in zinc measurements for the witness-line and the variation in zinc measurements for the intersection differ at $\alpha = .05$.

c. The largest sample variance is $s_2^2 = .1015^2$ and the smallest sample variance is $s_3^2 = .0443^2$. Since the test indicated there was no difference between the variances of the two populations, we can infer that there is no difference among all three variances.

d. The conditions necessary for the test results to be valid are:

1. The two sampled populations are normally distributed.
2. The samples are randomly and independently selected from their respective populations.

Since there are so few observations per group, it is very difficult to check these assumptions.

9.104 Since the printout represents a two-tailed test, we will first test to see if the two population variances are different. We will then see if the bid price variance for the surrounding market exceeds the bid price variance for the Tri-county market.

To determine if the bid price variance for the surrounding market exceeds the bid price variance for the Tri-county market, we test:

H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

The test statistic is $F = 1.41$ and the p-value is p-value = 0.048.

Since no α value was given, we will use $\alpha = .05$. Since the p-value is less than α (p-value = .048 < .05), H_0 is rejected. There is sufficient evidence to indicate a difference in the bid price variances for the two groups. Since the sample variance for the surrounding market is larger than the sample variance for the Tri-county market, we can conclude that the population bid price variance for the surrounding market exceeds that for the Tri-county market at $\alpha = .05$.

9.106 a. The 2 samples are randomly selected in an independent manner from the two populations. The sample sizes, n_1 and n_2, are large enough so that $\bar{x}_1$ and $\bar{x}_2$ each have approximately normal sampling distributions and so that s_1^2 and s_2^2 provide good approximations to σ_1^2 and σ_2^2. This will be true if $n_1 \geq 30$ and $n_2 \geq 30$.

b. 1. Both sampled populations have relative frequency distributions that are approximately normal.
2. The population variances are equal.
3. The samples are randomly and independently selected from the populations.

c. 1. The relative frequency distribution of the population of differences is normal.
2. The sample of differences is randomly selected from the population of differences.

d. The two samples are independent random samples from binomial distributions. Both samples should be large enough so that the normal distribution provides an adequate approximation to the sampling distributions of $\hat{p}_1$ and $\hat{p}_2$.

e. The two samples are independent random samples from populations which are normally distributed.

9.108 a. H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_2^2}{s_1^2} = \dfrac{120.1}{31.3} = 3.84$

The rejection region requires $\alpha/2 = .05/2 = .025$ in the upper tail of the F distribution with numerator df $\nu_1 = n_2 - 1 = 15 - 1 = 14$ and denominator df $\nu_2 = n_1 - 1 = 20 - 1 = 19$. From Table X, Appendix A, $F_{.025} \approx 2.66$. The rejection region is $F > 2.66$.

Since the observed value of the test statistic falls in the rejection region ($F = 3.84 > 2.66$), H_0 is rejected. There is sufficient evidence to conclude $\sigma_1^2 \neq \sigma_2^2$ at $\alpha = .05$.

b. No, we should not use a t test to test H_0: $(\mu_1 - \mu_2) = 0$ against H_a: $(\mu_1 - \mu_2) \neq 0$ because the assumption of equal variances does not seem to hold since we concluded $\sigma_1^2 \neq \sigma_2^2$ in part **a**.

9.110 Some preliminary calculations are:

$$\hat{p} = \frac{x_1}{n_1} = \frac{110}{200} = .55; \quad \hat{p}_2 = \frac{x_2}{n_2} = \frac{130}{200} = .65; \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{110 + 130}{200 + 200} = \frac{240}{400} = .60$$

a. H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 < 0$

The test statistic is $z = \dfrac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{(.55 - .65) - 0}{\sqrt{.6(1-.6)\left(\dfrac{1}{200} + \dfrac{1}{200}\right)}} = \dfrac{-.10}{.049} = -2.04$

The rejection region requires $\alpha = .10$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = -2.04 < -1.28$), H_0 is rejected. There is sufficient evidence to conclude ($p_1 - p_2 < 0$) at $\alpha = .10$.

b. For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The 95% confidence interval for $(p_1 - p_2)$ is approximately:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \Rightarrow (.55 - .65) \pm 1.96\sqrt{\frac{.55(1-.55)}{200} + \frac{.65(1-.65)}{200}}$$
$$\Rightarrow -.10 \pm .096 \Rightarrow (-.196, -.004)$$

c. From part **b**, $z_{.025} = 1.96$. Using the information from our samples, we can use $p_1 = .55$ and $p_2 = .65$. For a width of .01, the bound is .005.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2 (p_1 q_1 + p_2 q_2)}{(SE)^2} = \frac{(1.96)^2\left(.55(1-.55)+.65(1-.65)\right)}{.005^2}$$

$$= \frac{1.82476}{.000025} = 72{,}990.4 \approx 72{,}991$$

9.112 a. This is a paired difference experiment.

Pair	Difference (Pop. 1 – Pop. 2)
1	6
2	4
3	4
4	3
5	2

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{19}{5} = 3.8 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{81 - \frac{19^2}{5}}{5-1} = 2.2$$

$$s_d = \sqrt{2.2} = 1.4832$$

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{3.8-0}{1.4832/\sqrt{5}} = 5.73$

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the t distribution with df $= n_d - 1 = 5 - 1 = 4$. From Table VI, Appendix A, $t_{.025} = 2.776$. The rejection region is $t < -2.776$ or $t > 2.776$.

Since the observed value of the test statistic falls in the rejection region ($t = 5.73 > 2.776$), H_0 is rejected. There is sufficient evidence to indicate that the population means are different at $\alpha = .05$.

b. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. Therefore, we would use the same t value as above, $t_{.025} = 2.776$. The confidence interval is:

$$\bar{x}_d \pm t_{\alpha/2} \frac{s_d}{\sqrt{n_d}} \Rightarrow 3.8 \pm 2.776 \frac{1.4832}{\sqrt{5}} \Rightarrow 3.8 \pm 1.84 \Rightarrow (1.96,\ 5.64)$$

c. The sample of differences must be randomly selected from a population of differences which has a normal distribution.

9.114 a. For confidence coefficient .90, $\alpha = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The confidence interval is:

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \Rightarrow \left(39.08 - 38.79\right) \pm 1.645\sqrt{\frac{6.73^2}{127} + \frac{6.94^2}{114}}$$
$$\Rightarrow 0.29 \pm 1.452 \Rightarrow (-1.162,\ \ 1.742)$$

We are 90% confident that the true difference in the mean service-rating scores for male and female guests at Jamaican 5-star hotels is between -1.162 and 1.742.

b. Since 0 is contained in the 90% confidence interval, there is insufficient evidence to indicate a difference the perception of service quality at 5-star hotels in Jamaica between the genders.

c. To determine if the variances of guest scores for males and females differ, we test:

H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_2^2}{s_1^2} = \dfrac{6.94^2}{6.73^2} = 1.063$

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in the upper tail of the F distribution with numerator df $\nu_1 = n_2 - 1 = 114 - 1 = 113$ and denominator df $\nu_2 = n_1 - 1 = 127 - 1 = 126$. From Table IX, Appendix A, $F_{.05} \approx 1.26$. The rejection region is $F > 1.26$.

Since the observed value of the test statistic does not fall in the rejection region (F = 1.063 $\ngtr$ 1.26), H_0 is not rejected. There is insufficient evidence to indicate that the variances of guest scores for males and females differ at $\alpha = .10$.

9.116 a. Let p_1 = dropout rate for the varied exercise group and p_2 = dropout rate for the 'no set schedule' exercise group.

The observed dropout rate for the varied exercise group is $\hat{p}_1 = \dfrac{x_1}{n_1} = \dfrac{15}{40} = .375$

The observed dropout rate for the varied exercise group is $\hat{p}_2 = \dfrac{x_2}{n_2} = \dfrac{23}{40} = .575$.

b. For confidence coefficient .90, $\alpha = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The 90% confidence interval is:

$$\left(\hat{p}_1-\hat{p}_2\right)\pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}\Rightarrow(.375-.575)\pm 1.645\sqrt{\frac{.375(.625)}{40}+\frac{.575(.425)}{40}}$$
$$\Rightarrow -.200\pm .180\Rightarrow(-.380,\,-.020)$$

c. We are 90% confident that the difference between the dropout rates of the two groups of exercisers is between −.380 and −.020. Since both end points of the confidence interval are less than 0, there is evidence that the dropout rate for the varied exercise groups is less than the dropout rate for the "no set schedule" exercise group.

d. For confidence coefficient .90, $\alpha=.10$ and $\alpha/2=.10/2=.05$. From Table IV, Appendix A, $z_{.05}$ = 1.645.

$$n_1=n_2=\frac{\left(z_{\alpha/2}\right)^2(p_1q_1+p_2q_2)^2}{(SE)^2}=\frac{(1.645)^2\left(.4(.6)+.6(.4)\right)}{.1^2}=129.889\approx 130$$

9.118 a. Let σ_1^2 = variance in degree of swelling for mice treated with bear bile and σ_2^2 = variance in degree of swelling for mice treated with pig bile.

To determine if the variation in degree of swelling for mice treated with bear and pig bile differ, we test:

H_0: $\sigma_1^2=\sigma_2^2$
H_a: $\sigma_1^2=\sigma_2^2$

The test statistic is $F=\dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}}=\dfrac{s_1^2}{s_2^2}=\dfrac{4.17^2}{3.33^2}=1.568$

The rejection region requires $\alpha/2=.05/2=.025$ in the upper tail of the F distribution with $\nu_1=n_1-1=10-1=9$ and $\nu_2=n_2-1=10-1=9$. Using Table X, Appendix A, $F_{.025}$ = 4.03. The rejection region is $F > 4.03$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 1.568 \ngtr 4.03$), H_0 is not rejected. There is insufficient evidence to indicate the variation in degree of swelling for mice treated with bear and pig bile differ at $\alpha=.05$.

b. The conditions necessary for the test results to be valid are:

1. The two sampled populations are normally distributed.
2. The samples are randomly and independently selected from their respective populations.

9.120 For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha / 2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$.

An estimate of σ_1 and σ_2 is obtained from:

$$\text{range} \approx 4s \quad \text{Thus, } s \approx \frac{\text{range}}{4} = \frac{4}{4} = 1$$

$$n_1 = n_2 = \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{(SE)^2} = \frac{(1.96)^2(1^2 + 1^2)}{.2^2} = 192.08 \approx 193$$

You need to take 193 measurements at each site.

9.122 a. Since the data were collected as paired data, they must be analyzed as paired data. Since all winners were matched with a non-winner, the data are not independent.

b. Let μ_1 = mean life expectancy of Academy Award winners and μ_2 = mean life expectancy of non-winners. Then $\mu_d = \mu_1 - \mu_2$.

To compare the mean life expectancies of Academy Award winners and non-winners, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

c. From the information given, $p = .003$. Since the p-value is so small, we reject H_0. There is sufficient evidence to indicate that the mean life expectancies of Academy Award winners and non-winners differ at $\alpha > .003$.

9.124 a. Let μ_1 = mean effective population size for outcrossing snails and μ_2 = mean effective population size for selfing snails.

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(17-1)1932^2 + (5-1)1890^2}{17 + 5 - 2} = 3{,}700{,}519.2$$

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table VI, Appendix A, with df = $n_1 + n_2 - 2 = 17 + 5 - 2 = 20$, $t_{.05} = 1.725$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.05}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \Rightarrow (4{,}894 - 4{,}133) \pm 1.725\sqrt{3{,}700{,}519.2\left(\frac{1}{17} + \frac{1}{5}\right)}$$

$$\Rightarrow 761 \pm 1{,}688.19 \Rightarrow (-927.19,\ 2{,}449.19)$$

We are 90% confident that the difference in the mean effective population sizes for outcrossing snails and selfing snails is between –927.19 and 2,449.19.

b. Let σ_1^2 = variance of the effective population size of the outcrossing snails and σ_2^2 = variance of the effective population size of the selfing snails. To determine if the variances for the two groups differ, we test:

H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_1^2}{s_2^2} = \dfrac{1{,}932^2}{1{,}890^2} = 1.045$

Since α is not given, we will use $\alpha = .05$. The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in the upper tail of the F distribution with $\nu_1 = n_1 - 1 = 17 - 1 = 16$ and $\nu_2 = n_2 - 1 = 5 - 1 = 4$. From Table X, Appendix A, $F_{.025} \approx 8.66$. The rejection region is $F > 8.66$.

Since the observed value of the test statistic does not fall in the rejection region (F = 1.045 $\not> 8.66$), H_0 is not rejected. There is insufficient evidence to indicate the variances for the two groups differ at $\alpha = .05$.

9.126 Some preliminary calculations:

Twin A	Twin B	Diff-A-B		Twin A	Twin B	Diff-A-B
113	109	4		100	88	12
94	100	–6		100	104	–4
99	86	13		93	84	9
77	80	–3		99	95	4
81	95	–14		109	98	11
91	106	–15		95	100	–5
111	117	–6		75	86	–11
104	107	–3		104	103	1
85	85	0		73	78	–5
66	84	–18		88	99	–11
111	125	–14		92	111	–19
51	66	–15		108	110	–2
109	108	1		88	83	5
122	121	1		90	82	8
97	98	–1		79	76	3
82	94	–12		97	98	–1

Since the data were collected as paired data, we must analyze it using a paired t-test.

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{-93}{32} = -2.906 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{2723 - \frac{(-93)^2}{32}}{32 - 1} = 79.1200$$

$$s_d = \sqrt{s_d^2} = \sqrt{79.1200} = 8.895$$

To determine if there is a difference between the average IQ scores of identical twins, where one member of the pair is reared by the natural parents and the other member of the pair is not, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $z = \dfrac{\bar{x}_d - \mu_d}{s_d / \sqrt{n_d}} = \dfrac{-2.906 - 0}{8.895 / \sqrt{32}} = -1.848$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.848 \not< -1.96$), H_0 is not rejected. There is insufficient evidence to indicate there is a difference between the average IQ scores of identical twins, where one member of the pair is reared by the natural parents and the other member of the pair is not at $\alpha = .05$.

9.128 Let μ_1 = mean impulsive-sensation seeking score for cocaine abusers and μ_2 = mean impulsive-sensation seeking score for college students.

To determine if there is a difference in the mean impulsive-sensation scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(9.4 - 9.5) - 0}{\sqrt{\dfrac{4.4^2}{450} + \dfrac{4.4^2}{589}}} = -.36$

The rejection region requires $\alpha / 2 = .01 / 2 = .005$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.005} = 2.58$. The rejection region is $z < -2.58$ or $z > 2.58$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -.36 \not< -2.58$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean impulsive-sensation scores for the two groups at $\alpha = .01$.

Let μ_1 = mean sociability score for cocaine abusers and μ_2 = mean sociability score for college students.

To determine if there is a difference in the mean sociability scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(10.4 - 12.5) - 0}{\sqrt{\dfrac{4.3^2}{450} + \dfrac{4.0^2}{589}}} = -8.04$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic falls in the rejection region ($z = -8.04 < -2.58$), H_0 is rejected. There is sufficient evidence to indicate a difference in the mean sociability scores for the two groups at $\alpha = .01$.

Let μ_1 = mean neuroticism-anxiety score for cocaine abusers and μ_2 = mean neuroticism-anxiety score for college students.

To determine if there is a difference in the mean neuroticism-anxiety scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(8.6 - 9.1) - 0}{\sqrt{\dfrac{5.1^2}{450} + \dfrac{4.6^2}{589}}} = -1.63$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.63 \nless -2.58$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean neuroticism-anxiety scores for the two groups at $\alpha = .01$.

Let μ_1 = mean aggression-hostility score for cocaine abusers and μ_2 = mean aggression-hostility score for college students.

To determine if there is a difference in the mean aggression-hostility scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(8.6 - 7.3) - 0}{\sqrt{\dfrac{3.9^2}{450} + \dfrac{4.1^2}{589}}} = 5.21$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic falls in the rejection region ($z = 5.21 > 2.58$), H_0 is rejected. There is sufficient evidence to indicate a difference in the mean aggression-hostility scores for the two groups at $\alpha = .01$.

Let μ_1 = mean activity score for cocaine abusers and μ_2 = mean activity score for college students.

To determine if there is a difference in the mean activity scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(11.1 - 8.0) - 0}{\sqrt{\dfrac{3.4^2}{450} + \dfrac{4.1^2}{589}}} = 13.31$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic falls in the rejection region ($z = 13.31 > 2.58$), H_0 is rejected. There is sufficient evidence to indicate a difference in the mean activity scores for the two groups at $\alpha = .01$.

9.130 Let μ_1 = mean number of swims by male rat pups and μ_2 = mean number of swims by female rat pups. Then $\mu_d = \mu_1 - \mu_2$.

Some preliminary calculations are:

Litter	Male	Female	Diff	Litter	Male	Female	Diff
1	8	5	3	11	6	5	1
2	8	4	4	12	6	3	3
3	6	7	-1	13	12	5	7
4	6	3	3	14	3	8	-5
5	6	5	1	15	3	4	-1
6	6	3	3	16	8	12	-4
7	3	8	-5	17	3	6	-3
8	5	10	-5	18	6	4	2
9	4	4	0	19	9	5	4
10	4	4	0				

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{7}{19} = .368 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{225 - \frac{7^2}{19}}{19 - 1} = \frac{222.4211}{18} = 12.3567$$

$$s_d = \sqrt{12.3567} = 3.5152$$

To determine if there is a difference in the mean number of swims required by male and female rat pups, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{\frac{s_d}{\sqrt{n_d}}} = \dfrac{.368 - 0}{\frac{3.5152}{\sqrt{19}}} = .456$

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in each tail of the t distribution with df = n_d -1 = 19 – 1 = 18. From Table IV, Appendix A, $t_{.05}$ = 1.734. The rejection region is $t > 1.734$ or $t < -1.734$.

Since the test statistic does not fall in the rejection region $(t = .456 \not> 1.734)$, H_0 is not rejected. There is insufficient evidence to indicate there is a difference in the mean number of swims required by male and female rat pups at $\alpha = .10$.

The differences are probably not from a normal distribution.

9.132 a. Let μ_1 = mean size of the right temporal lobe of the brains for the short-recovery group and μ_2 = mean size of the right temporal lobe of the brains for the long-recovery group.

The parameter of interest is $\mu_1 - \mu_2$.

The necessary assumptions are:

1. Both populations being sampled from are normal.
2. The variances of the two populations are the same.
3. The samples are random and independent.

b. Let p_1 = proportion of athletes who have a good self-image of their body and p_2 = proportion of non-athletes who have a good self-image of their body.

The parameter of interest is $p_1 - p_2$.

The necessary assumptions are:

1. The two samples are independent and random.
2. Both samples are large enough that the normal distribution provides an adequate approximation to the sampling distributions of $\hat{p}_1$ and $\hat{p}_2$.

c. Let μ_1 = mean weight of eggs produced by chickens on regular feed and μ_2 = mean weight of eggs produced by chickens on corn oil.

The parameter of interest is $\mu_d = \mu_1 - \mu_2$.

If the total number of sampled chickens is less than 30, the necessary assumptions are:

1. The population of differences is normal.
2. The sample of differences is random.

If the total number of sampled chickens is greater than or equal to 30, the necessary assumption is:

1. The sample of differences is random.

9.134 a. Let μ_1 = mean compression ratio for the standard method and μ_2 = mean compression ratio for the Huffman-coding method. The target parameter is $\mu_d = \mu_1 - \mu_2$, the difference in the mean compression ratios for the standard method and the Hoffman-coding method.

Some preliminary calculations are:

Circuit	**Standard Method**	**Huffman-coding Method**	**Difference**
1	.80	.78	.02
2	.80	.80	0
3	.83	.86	-.03
4	.53	.53	0
5	.50	.51	-.01
6	.96	.68	.28
7	.99	.82	.17
8	.98	.72	.26
9	.81	.45	.36
10	.95	.79	.16
11	.99	.77	.22

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{1.43}{11} = .13 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{.3799 - \frac{1.43^2}{11}}{11-1} = \frac{.194}{10} = .0194$$

$s_d = \sqrt{.0194} = .1393$

For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table VI, Appendix A, with df = n_d – 1 = 11 – 1 = 10, $t_{.025}$ = 2.228. The 95% confidence interval is:

$$\bar{x}_d \pm t_{.025}\frac{s_d}{\sqrt{n_d}} \Rightarrow 0.13 \pm 2.228\frac{.1393}{\sqrt{11}} \Rightarrow 0.13 \pm 0.094 \Rightarrow (0.036, \quad 0.224)$$

We are 95% confident that the difference in the mean compression ratios between the standard method and the Hoffman-coding method is between .036 and .224. Since all of the values in the confidence interval are positive, the Hoffman-coding method has the smaller compression ratio.

b. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. The standard error is .03.

$$n_d = \frac{\left(z_{\alpha/2}\right)^2 \sigma_d^2}{(SE)^2} = \frac{1.96^2(.1393)^2}{.03^2} = 82.8 \approx 83$$

9.136 a. Let μ_1 = mean accuracy score for sketching a world map for the group receiving the mental map lessons and μ_2 = mean accuracy score for sketching a world map for the group receiving traditional instruction.

To determine if the mental map lessons improve a student's ability to sketch a world map, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 > 0$

b. Let μ_1 = mean accuracy score for continents drawn for the group receiving the mental map lessons and μ_2 = mean accuracy score for continents drawn for the group receiving traditional instruction.

Suppose we test the hypotheses using $\alpha = .05$. Since the observed p-value (.0507) for the test is not less than $\alpha = .05$, there is no evidence to reject H_0. There is insufficient evidence to indicate that the mental map lessons improve a student's ability to draw continents at $\alpha = .05$.

c. Let μ_1 = mean accuracy score for labeling oceans for the group receiving the mental map lessons and μ_2 = mean accuracy score for labeling oceans for the group receiving traditional instruction.

Suppose we test the hypotheses using $\alpha = .05$. Since the observed p-value (.7371) for the test is not less than $\alpha = .05$, there is no evidence to reject H_0. There is insufficient evidence to indicate that the mental map lessons improve a student's ability for labeling oceans at $\alpha = .05$.

d. Let μ_1 = mean accuracy score for the entire map for the group receiving the mental map lessons and μ_2 = mean accuracy score for the entire map for the group receiving traditional instruction.
Suppose we test the hypotheses using $\alpha = .05$. Since the observed p-value (.0024) for the test is less than $\alpha = .05$, there is evidence to reject H_0. There is sufficient evidence to indicate that the mental map lessons improve a student's ability for identifying the entire map at $\alpha = .05$.

e. We must assume that the populations of accuracy scores for each group are normally distributed, the samples are independent, and the variances of the two groups are the same. The assumption that the populations of accuracy scores are normally distributed is probably not valid. The accuracies of the sketches were evaluated on a scale from 1 to 5. It is very unlikely that the scores are normally distributed when there are only 5 possible values.

9.138 Let μ_1 = mean ingestion time for green blades and μ_2 = mean ingestion time for decayed blades.

$$s_p^2 = \frac{(n_1-1)s_1^2+(n_1-1)s_2^2}{n_1+n_2-2} = \frac{(10-1).79^2+(10-1).47^2}{10+10-2} = .4225$$

Since no α was given, we will use $\alpha = .10$. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table VI, Appendix A, with df = $n_1 + n_2 - 2 = 10 + 10 - 2 = 18$, $t_{.05} = 1.734$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.05}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} \Rightarrow (3.35-2.36) \pm 1.734\sqrt{.4225\left(\frac{1}{10}+\frac{1}{10}\right)}$$

$$\Rightarrow .99 \pm .504 \Rightarrow (.486, 1.494)$$

We are 90% confident that the difference in the mean ingestion times for the two types of leaves is between .486 and 1.494 hours.

Yes, this supports the conclusions. Since 0 is not in the 90% confidence interval, there is evidence to indicate that the mean ingestion times for the two types of blades are different. Since all of the values in the interval are positive, the mean ingestion time for the decayed blades is faster than that for the green blades.

9.140 Using MINITAB, the descriptive statistics are:

Descriptive Statistics: AgeDiff, HtDiff, WtDiff

```
Variable   N   Mean  StDev  Minimum      Q1  Median    Q3  Maximum
AgeDiff   10  0.800   3.88    -8.00  -0.500    1.00  3.25     6.00
HtDiff    10   1.38   3.23    -3.60   -1.25   0.950  4.05     6.20
WtDiff    10  -2.83   5.67   -16.10   -5.17   -2.35  1.47     4.50
```

(Note: The differences were computed by taking the measurement for those with MS – the measurement for those without MS.)

To determine if there is a difference in mean age between those with MS and those without, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \frac{\bar{x}_d - 0}{s_d/\sqrt{n_d}} = \frac{0.8-0}{3.88/\sqrt{10}} = 0.65$

We will use $\alpha = .05$ for all tests. The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the t distribution with df = $n_d - 1 = 10 - 1 = 9$. From Table VI, Appendix A, $t_{.025} = 2.262$. The rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 0.65 \not> 2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean age between those with MS and those without at $\alpha = .05$.

To determine if there is a difference in mean height between those with MS and those without, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{1.38 - 0}{3.23 / \sqrt{10}} = 1.35$

From above, the rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.35 \not> 2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean height between those with MS and those without at $\alpha = .05$.

To determine if there is a difference in mean weight between those with MS and those without, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{-2.83 - 0}{5.67 / \sqrt{10}} = -1.58$

From above, the rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.58 \not< -2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean weight between those with MS and those without at $\alpha = .05$.

Thus, it appears that the researchers have successfully matched the MS and non-MS subjects.