Objectives

1. Add or subtract like radicals.
2. Use the distributive property in expressions containing radicals.
3. Simplify radical expressions that contain mixed operations.

Vocabulary

Like Radicals — Radical expressions with identical radicands and identical indexes.

Prior Knowledge

Combining like terms.

New Concepts

When we have an algebraic expression like 2x + 3x, we can combine them to equal 5x because the variables are exactly the same. Likewise, if we have exactly the same radical, we can combine like radicals.

$$2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$$

We must have exactly the same **radicand** and the same **index**.

$$3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$$

Example 1: Simplify $3y^3\sqrt[4]{8y} - 9y^3\sqrt[4]{8y}$

$-6y^3\sqrt[4]{8y}$	Because the radicals are the same, combine the coefficients.

To add radicals that are not the same, attempt to simplify them so they become like radicals. We do NOT use decimal approximations. Leave the answers in exact form.

Example 2: Simplify $3\sqrt{24} + \sqrt{54}$

$3\sqrt{4 \cdot 6} + \sqrt{9 \cdot 6}$	Rewrite each radicand with perfect square factors.
$3 \cdot 2\sqrt{6} + 3\sqrt{6}$	Simplify the perfect square factors.
$6\sqrt{6} + 3\sqrt{6}$	Multiply factors.
$9\sqrt{6}$	Because the radicals are the same, combine the coefficients.

Example 3: Simplify $\sqrt{27x} + \sqrt{75x}$

$\sqrt{9 \cdot 3x} + \sqrt{25 \cdot 3x}$	Rewrite each radicand with perfect square factors.
$3\sqrt{3x} + 5\sqrt{3x}$	Simplify the perfect square factors.
$8\sqrt{3x}$	Because the radicals are the same, combine the coefficients.

Example 4: Simplify $4\sqrt{98y^5} - 7\sqrt{128y^5}$

$4\sqrt{49 \cdot 2y^4 y} - 7\sqrt{64 \cdot 2y^4 y}$	Rewrite each radicand with perfect square factors.
$4 \cdot 7y^2\sqrt{2y} - 7 \cdot 8y^2\sqrt{2y}$	Simplify the perfect square factors.
$28y^2\sqrt{2y} - 56y^2\sqrt{2y}$	Multiply factors.
$-28y^2\sqrt{2y}$	Because the radicals are the same, combine the coefficients.

Example 5: Simplify $6y\sqrt[4]{81y^5} - 2y\sqrt[4]{16y^6}$

$6y \cdot 3y\sqrt[4]{y} - 2y \cdot 2y^2\sqrt[4]{y^2}$	Simplify the perfect square factors.
$18y^2\sqrt[4]{y} - 4y^4\sqrt[4]{y^2}$	Multiply factors. The radicals are not the same so the terms can NOT be combined.

2. Use the distributive property in expressions containing radicals. Simplify radical expressions that contain mixed operations.

All of the multiplication skills that we have used previously still apply. Notice the similarities in the following problems.

$2(3x + 4)$ $= 6x + 8$	$\sqrt{2}\left(\sqrt{32} - \sqrt{6}\right)$ $= \sqrt{64} - \sqrt{12}$ $= 8 - 2\sqrt{3}$	Distributive Property
$(3a - 2)(5a + 7)$ $= 15a^2 + 21a - 10a - 14$ $= 15a^2 + 11a - 14$	$\left(7 - \sqrt{3}\right)\left(\sqrt{5} + \sqrt{2}\right)$ $= 7\sqrt{5} + 7\sqrt{2} - \sqrt{15} - \sqrt{6}$	FOIL
$(5x - 3)^2$ $= (5x - 3)(5x - 3)$ $= 25x^2 - 15x - 15x + 9$ $= 25x^2 - 30x + 9$	$\left(\sqrt{7} - 3\right)^2$ $= \left(\sqrt{7} - 3\right)\left(\sqrt{7} - 3\right)$ $= \sqrt{49} - 3\sqrt{7} - 3\sqrt{7} + 9$ $= 7 - 6\sqrt{7} + 9$ $= 16 - 6\sqrt{7}$	Perfect Square
$(2x - 5)(2x + 5)$ $= 4x^2 + 10x - 10x - 25$ $= 4x^2 - 25$	$\left(\sqrt{3} - 2\right)\left(\sqrt{3} + 2\right)$ $= \sqrt{9} + 2\sqrt{3} - 2\sqrt{3} - 4$ $= 3 - 4$ $= -1$	Sum and Difference of Two Terms

To **multiply** radicals, the index must be the same. If the index is the same, multiply the coefficients together and multiply the radicands together.

To **add** radicals, the radicals must be the same (same index and same radicand). Add the coefficients together and keep the same radical.

Example 6: $\sqrt{7}\left(\sqrt{5}-3\sqrt{14}\right)$

$\sqrt{7}\cdot\sqrt{5}-\sqrt{7}\cdot 3\sqrt{14}$	Multiply each term in the parentheses by $\sqrt{7}$.
$\sqrt{35}-3\sqrt{98}$	The indexes are the same so multiply the coefficients and the radicands.
$\sqrt{35}-3\cdot 7\sqrt{2}$	Simplify each radical.
$\sqrt{35}-21\sqrt{2}$	The radicals are not the same so they CANNOT be combined.

Example 7: $\left(5-\sqrt{a}\right)\left(2+\sqrt{a}\right)$

$5\cdot 2+5\sqrt{a}-2\sqrt{a}-\sqrt{a}\cdot\sqrt{a}$	Foil.
$10+3\sqrt{a}-\sqrt{a^2}$	Multiply and simplify.
$10+3\sqrt{a}-a$	

Example 8: $\left(\sqrt[3]{9}+5\right)\left(\sqrt[3]{9}-2\right)$

$\sqrt[3]{9}\cdot\sqrt[3]{9}-2\sqrt[3]{9}+5\sqrt[3]{9}-10$	Foil.
$\sqrt[3]{81}+3\sqrt[3]{9}-10$	Multiply and simplify.
$3\sqrt[3]{3}+3\sqrt[3]{9}-10$	

Example 9: $\left(3+2\sqrt{5}\right)^2$

$\left(3+2\sqrt{5}\right)\left(3+2\sqrt{5}\right)$	Rewrite and foil.
$3\cdot 3+3\cdot 2\sqrt{5}+3\cdot 2\sqrt{5}+2\sqrt{5}\cdot 2\sqrt{5}$	
$9+6\sqrt{5}+6\sqrt{5}+4\sqrt{25}$	Multiply and simplify.
$9+12\sqrt{5}+4\cdot 5$	
$9+12\sqrt{5}+20$	
$29+12\sqrt{5}$	

Example 10: $\left(5+3\sqrt{3}\right)\left(5-3\sqrt{3}\right)$

$5\cdot 5-5\cdot 3\sqrt{3}+5\cdot 3\sqrt{3}-3\sqrt{3}\cdot 3\sqrt{3}$	Foil.
$25-15\sqrt{3}+15\sqrt{3}-9\sqrt{9}$	Multiply and simplify.
$25-9\cdot 3$	
$25-27$	
-2	